Computational methods in operations research
14 February, 2019

In the following exercises, you have to write up an LP/IP model for the given problem. Both the number of variables and number of constraints should be polynomial in the size of the input.

Exercise 1. Given a polyhedron $P = \{x \in \mathbb{R}^n : Ax \leq b\}$ and a vector $a \in \mathbb{R}^n$, find a point in P for which

(a) $\| x - a \|_\infty := \max |x_i - a_i|$ is minimum;

(b) $\| x - a \|_1 := \sum_{i_1}^n |x_i - a_i|$ is minimum.

Exercise 2. Let $P = \{x \in \mathbb{R}^n : Ax = b, x \geq 0\} \neq \emptyset$ a bounded polyhedron, and assume that for every $x \in P$ we have $dx + d_0 > 0$ for $d_0 \in \mathbb{R}^n$ and $d_0 \in \mathbb{R}$. Find an optimum solution for the following problem:

$$\max \frac{cx + c_0}{dx + d_0}, \; x \in P.$$

Exercise 3. Formalize the traveling salesman problem as an integer program.

Exercise 4. Formalize the maximum cut problem as an integer program.

Exercise 5. We are given an undirected graph $G = (V, E)$, a node $s \in V$ and a weight function $w : E \to \mathbb{R}$. Two spanning trees are called independent if for every $v \in V$, the two paths from s to v determined by the trees are node-disjoint (apart from s and v, of course). Find a pair of independent spanning trees with minimum total weight.